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ABSTRACT

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The problem of motion in a forceless field of the two parts, formed after a gas tank bursts in a vacuum, is considered. Assuming isothermal gas expansion, the differential equation for the motion of the two gas-tank parts is similar to the Blasius' boundary layer equation. For the adiabatic expansion, the equation of motion is:

$$f''' + ff''^{n} = 0 \quad \left(n = \frac{3\varkappa - 1}{2\varkappa}\right),$$

where $\varkappa$ - adiabatic index.

The solutions of the equations for both cases are tabulated for $\varkappa$ = 1; 1.25; 1.4.

This article solves the problem of the motion in a forceless field of two parts formed after a tank filled with gas bursts in a vacuum. /1*

1. Let us examine an arbitrary tank with the volume V, which is filled with gas in a forceless field having the following parameters: pressure P_0, temperature T_0, adiabatic index $\varkappa$, and gas constant R.

Author

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* Note: Numbers in the margin indicate pagination in the original foreign text.

At the time $\tau = 0$ the tank bursts in a plane perpendicular to the axis of the volume along the perimeter Π. The area of the transverse cross section of the tank in the burst plane is F. Without allowance for the mass of the gas which the parts contain - which can be assumed to be negligibly small as compared with the tank frame - both parts of the tank having the masses M_1 and M_2 begin to diverge from each other under the influence of the internal pressure. The gas from the internal tank cavity in a vacuum escapes through the gap which is formed. Let us determine the parameters /2 for the motion of the two tank parts.

We shall measure the coordinate x along the tank axis which is perpendicular to the burst plane, and we shall select the position of the burst plane at the time $\tau = 0$ as the origin. Let us examine the case when the gas escapes through the gap between the two tank parts in such a way that the component of the gas stream momentum on the x axis equals zero, and the axes of the equivalent pressure forces pass through the centers of mass of the corresponding tank parts. We shall assume that the velocity of the gas within the tank is small as compared with the velocity of the gas escape through the gap.

Under these assumptions, the equations of motion for two tank parts can be written as follows:

$$M_i \frac{d^2 x_i}{d\tau^2} = PF \qquad (i = 1, 2) \tag{1}$$

for

$$\tau = 0: \quad x_i = 0, \quad \frac{dx_i}{d\tau} = 0 \tag{2}$$

where x_i is the coordinate of the burst plane, and P - the variable gas pressure in the tank.

We can write the following for the relative motion of the tank parts:

$$\bar{M}\frac{d^2x}{d\tau^2} = PF \tag{3}$$

for $\tau = 0: \quad x = 0, \quad \frac{dx}{d\tau} = 0,$ /3

where $M = \frac{M_1 M_2}{M_1 + M_2}$ is the reduced mass, and $x = x_1 + x_2$.

Let us differentiate equation (3) with respect to τ:

$$\frac{d^3x}{d\tau^3} = \frac{F}{\bar{M}} \cdot \frac{dP}{d\tau} \tag{4}$$

for $\tau = 0: \; x = 0, \; \frac{dx}{d\tau} = 0, \; \frac{d^2x}{d\tau^2} = \frac{P_0 F}{\bar{M}}$

We can obtain the expression for P from equation (3):

$$P = \frac{\bar{M}}{F}\frac{d^2x}{d\tau^2} \tag{5}$$

In order to solve the equation of motion (4), it is necessary to have an additional relationship which determines the change in pressure P.

2. Let us examine the case when the escape process is isothermal due to heat exhange of the gas with the tank walls: $T = T_0 = \text{const}$.

Under the assumptions formulated above, the equation of state is valid for a gas located in a tank(1):

$$PV = \mathcal{G}RT_0 \tag{6}$$

where $\mathcal{G}$ is the total gas mass. The gas mass change in the tank is /4

$$\frac{d\mathcal{G}}{d\tau} = -G \tag{7}$$

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(1) We shall assume that when both parts diverge from each other, the tank volume during the process changes to a negligibly small extent, i.e. V = const.

where G is the mass discharge of gas through the gap, between two tank parts.

Let us write the expression for the gas discharge through the gap during critical escape in a vacuum with the stream contraction coefficient equal to K:

$$G = K \Pi x \rho_* a_* = K \Pi x \left(\frac{2}{\varkappa+1}\right)^{\frac{1}{\varkappa-1}} \frac{P}{RT_0} a_* \tag{8}$$

where a_* is the critical velocity,

ρ_* is the density in the critical cross section.

Taking into account (6), (7) and (8), we can write:

$$\frac{dP}{d\tau} = -\left(\frac{2}{\varkappa+1}\right)^{\frac{1}{\varkappa-1}} K \Pi a_* x \frac{P}{V} \tag{9}$$

Utilizing equations (5) and (9), we can rewrite (4) in the following form:

$$\frac{d^3x}{d\tau^3} + \left(\frac{2}{\varkappa+1}\right)^{\frac{1}{\varkappa-1}} \frac{K \Pi a_*}{V} x \frac{d^2x}{d\tau^2} = 0 \tag{10}$$

The initial conditions are: /5

$$\tau = 0: \; x = 0; \; \frac{dx}{d\tau} = 0, \; \frac{d^2x}{d\tau^2} = \frac{P_0 F}{\bar{M}} \tag{11}$$

Let us reduce (10) and the boundary conditions (11) to a dimensionless form. In order to do this, let us introduce the characteristic scales for the coordinate x —— X and for time τ —— θ, and let us make the following definition:

$$f = \frac{x}{X}, \quad \eta = \frac{\tau}{\theta} \tag{12}$$

Let us select the scales X and θ, so that in the first place the coefficient equals one in the case of $f \frac{d^2f}{d\eta^2}$, and in the second place $\left.\frac{d^2f}{d\eta^2}\right/_{\eta=0} = 1$. We then have:

$$X = \sqrt[3]{\left(\frac{P_0 F}{\bar{M}}\right)\left[\frac{V}{\left(\frac{2}{\varkappa+1}\right)^{\frac{1}{\varkappa-1}} K \Pi a_{*0}}\right]^2} \tag{13}$$

$$\Theta = \sqrt[3]{\left(\frac{\bar{M}}{P_0 F}\right)\left[\frac{V}{\left(\frac{2}{\varkappa+1}\right)^{\frac{1}{\varkappa-1}} K \Pi a_{*0}}\right]} \tag{14}$$

Thus, the problem under consideration can be reduced to the following form:

$$f''' + ff'' = 0 \tag{15}$$

with the initial conditions /6

$$\eta = 0: \quad f = 0, \quad f' = 0, \quad f'' = 1 \tag{16}$$

Under the initial conditions (16), equation (15) was solved numerically on an electronic computer.

The moment at which the gas ceases to influence both parts of the tank corresponds to values of η at which $f'' \simeq 0$.

We would like to note that a differential equation such as (15) is well known in hydrodynamics as the Blasius' boundary layer equation[1]. The results of a numerical solution for the equation which can be written in the following form are usually given in the boundary layer theory:

$$2\varphi''' + \varphi\varphi'' = 0 \tag{17}$$

under the conditions that

$$\zeta = 0: \quad \varphi = 0, \quad \varphi' = 0; \quad \zeta \to \infty: \varphi' = 1 \tag{18}$$

We can readily turn to the problem of (15) - (16) from the problem of

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(1) See, for example, G. Schlichting, "Boundary Layer Theory", Ivdatel'stvo Inostrannoy Literatury, 1956.

(17) - (18), with allowance for the fact that for $\zeta = 0$, $\varphi'' = 0.33206$:

$$f = \frac{1}{\sqrt[3]{4 \cdot 0{,}33206}}\,\varphi, \qquad \eta = \sqrt[3]{\frac{0{,}33206}{2}}\,\zeta \tag{19}$$

3. Let us examine the case of a thermally insulated tank, when the /7
process by which the gas escapes is adiabatic $\frac{P}{\rho^{\varkappa}} = \text{const}$, $\frac{P}{T^{\varkappa/\varkappa - 1}} = \text{const}$.

If we perform computations which are similar to those in section 2, substituting temperature T and density ρ by the pressure P using the adiabatic relationships, we obtain the following equation:

$$f''' + ff''^{n} = 0 \tag{20}$$

with the initial conditions:

$$\eta = 0: \quad f = 0, \quad f' = 0, \quad f'' = 1 \tag{21}$$

where

$$n = \frac{3\varkappa - 1}{2\varkappa}$$

The change from dimensionless coordinates (f,η) to real coordinates (x,τ) is performed according to the relationship

$$f = \frac{x}{\chi}, \quad \eta = \frac{\tau}{\theta}, \tag{22}$$

where

$$\chi = \sqrt[3]{\left(\frac{P_0 F}{\bar{M}}\right)\left[\frac{V}{\left(\frac{2}{\varkappa+1}\right)^{1/\varkappa - 1} K \Pi a_{*0}}\right]^2 \frac{1}{\varkappa^2}}$$

$$\Theta = \sqrt[3]{\left(\frac{\bar{M}}{P_0 F}\right)\left[\frac{V}{\left(\frac{2}{\varkappa+1}\right)^{1/\varkappa - 1} K \Pi a_{*0}}\right] \frac{1}{\varkappa}} \tag{23}$$

Here a_{*0} is the critical velocity at the initial moment of time $\tau = 0$ at the /8
temperature T_0. The expressions for χ and θ differ from the corresponding expressions in section 2 only by the presence of additional terms containing

the adiabatic index.

An electronic computer was used to perform a numerical solution of equation (20) with the boundary conditions (21) for the two values $\eta = 1.1$ and $\varkappa = 1.25$, and $\eta = 1.2$, $\varkappa = 1.667$. The computational results are presented in Figures 1, 2 and 3. A comparison of the results showed that the dimensionless parameters of motion differ insignificantly for the values of the adiabatic index under consideration.

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1. Schlichting, G. "Boundary Layer Theory". Ivdatel'stvo Inostrannoy Literatury, 1956.

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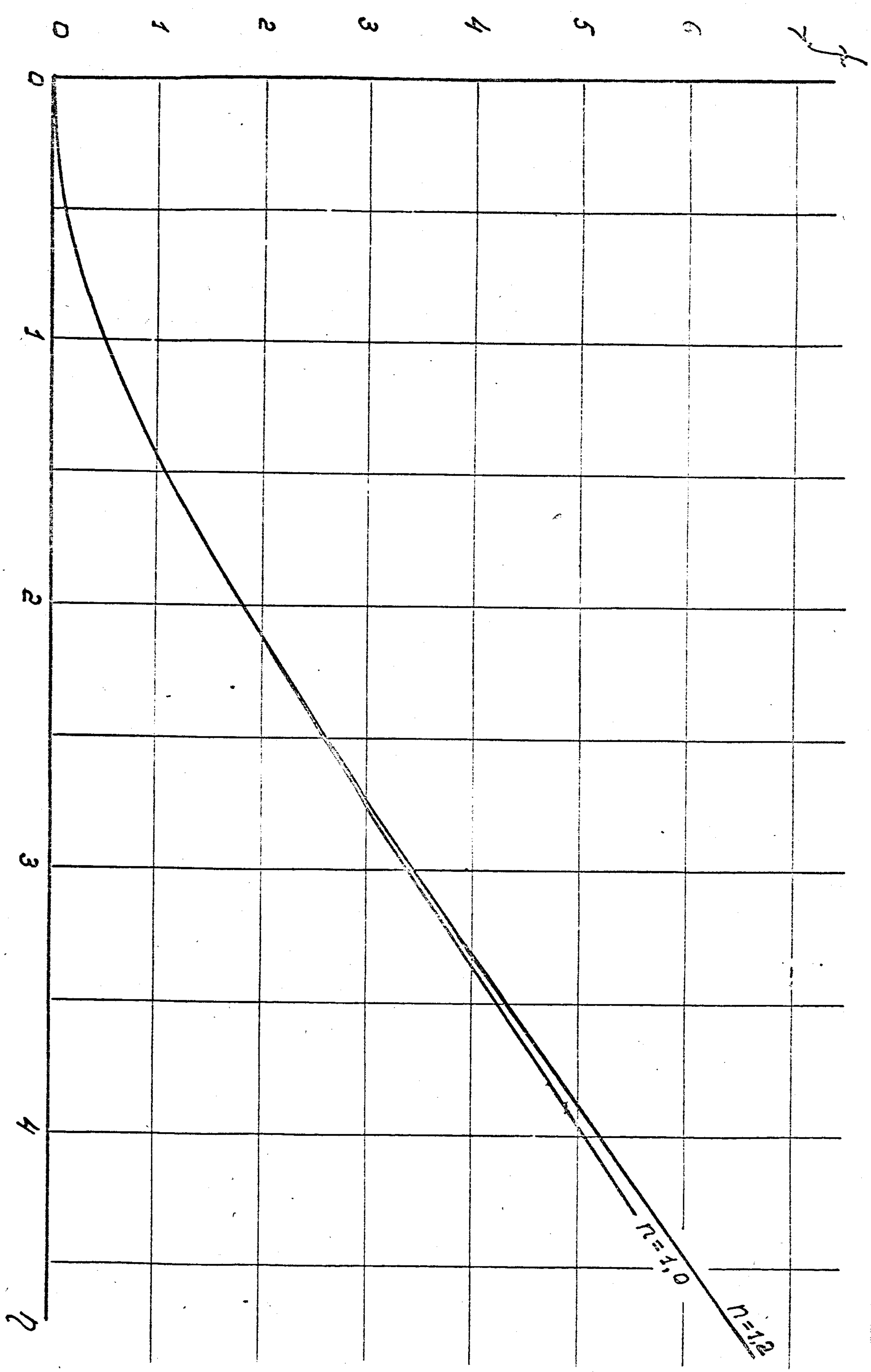

Figure 1

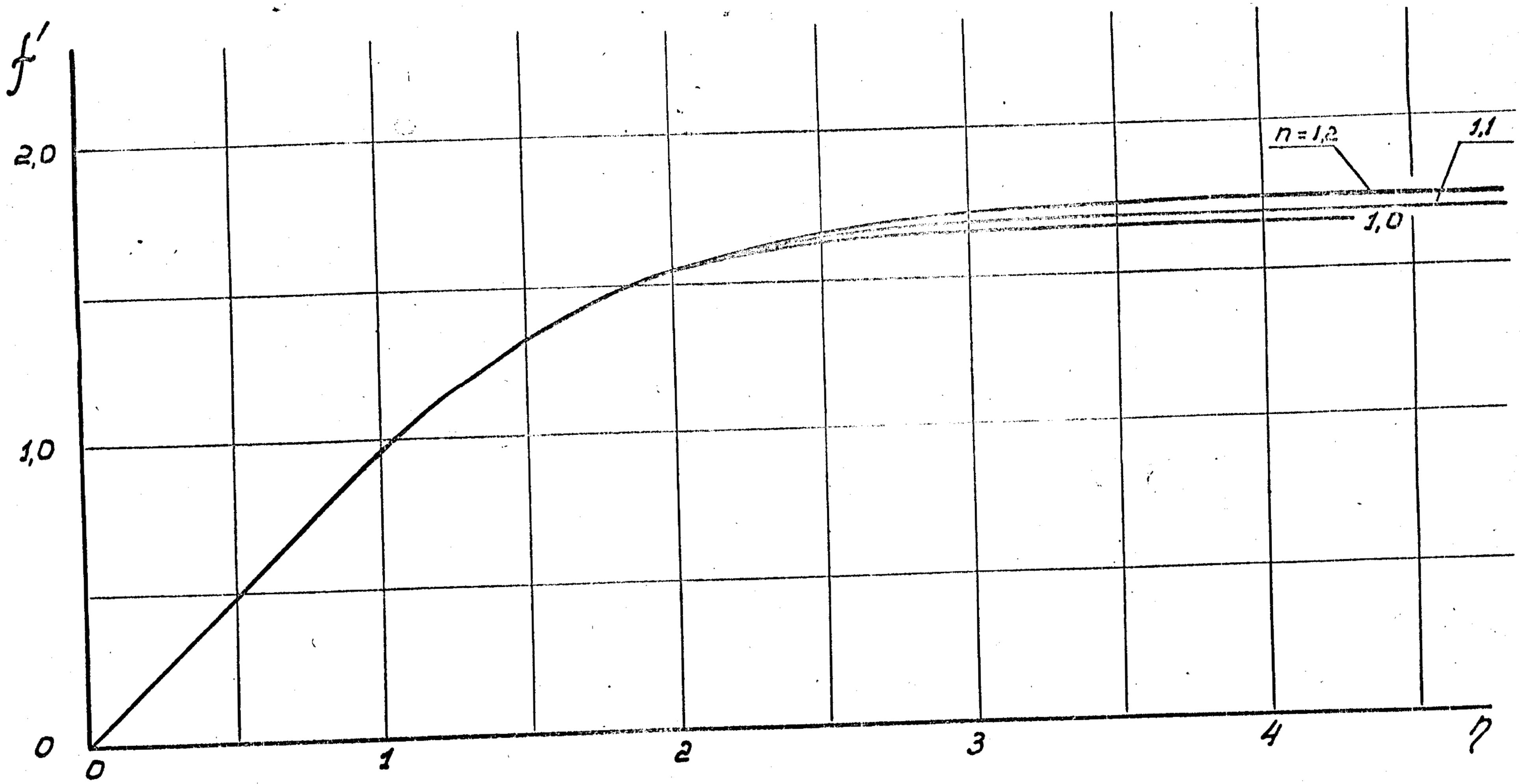

Figure 2

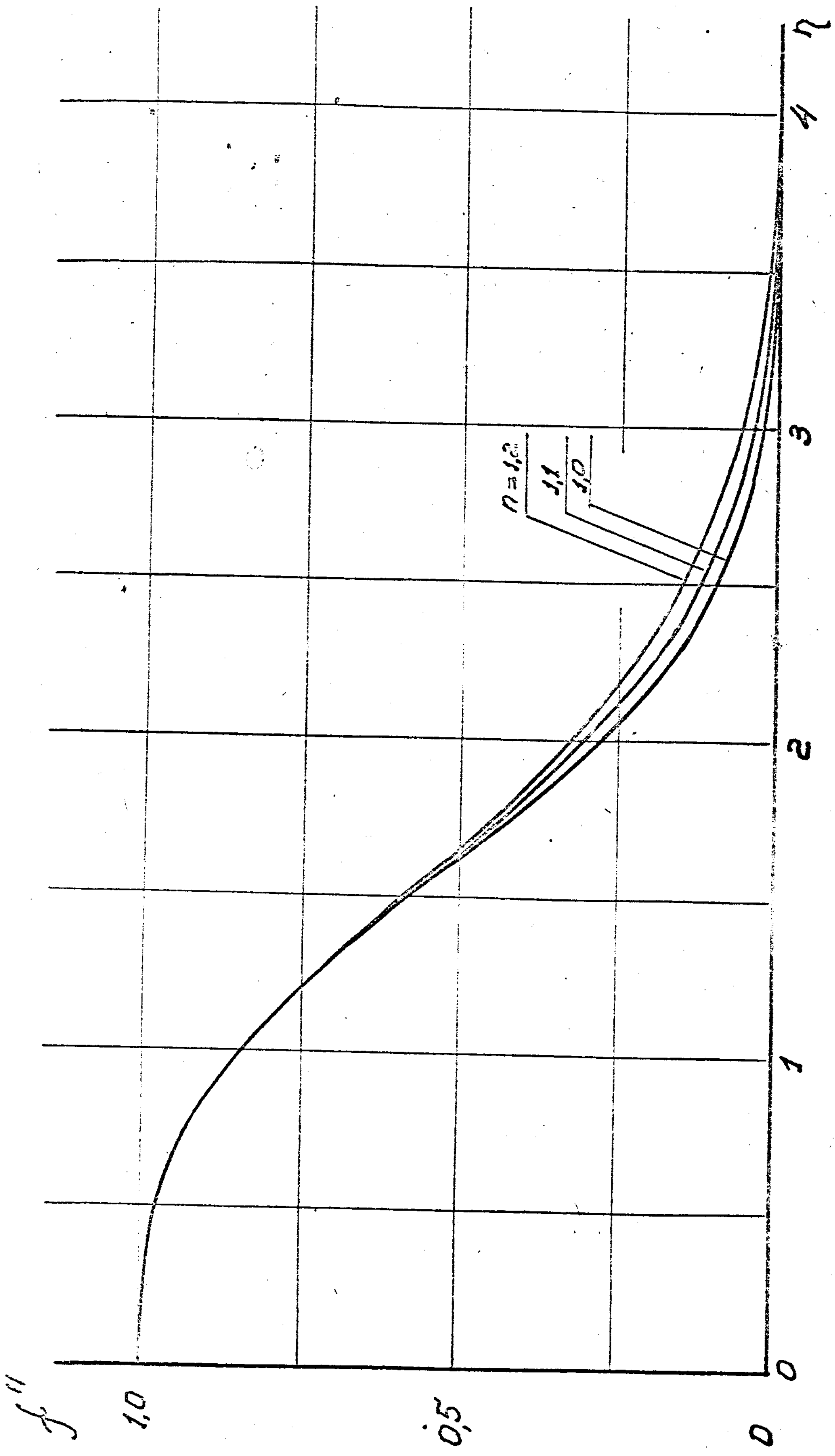

Figure 3